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Quiz 2

$f(x) = c\,(9-x^2)$, for $0 < x < 3$, find E(X), Var(X), F(X).

$c\int_0^3 (9-x^2)dx = 1 \Rightarrow c = 1/18$, $E(X) = \int_0^3 x(9-x^2)dx = 81/4$, $E(X^2) = \int_0^3 x^2(9-x^2)dx = 162/5$,

$Var(X) = E(X^2) - [E(X)]^2 = 162/5 - [1/18]^2$, $F(X) = \int_0^x \frac{(9-x^2)}{18}dx = \frac{27x - x^3}{54}$.

$f(x) = (1/x)$ for $1<x<e$, find E(lnX), Var(3X).

$E(\ln X) = \int_1^e \frac{Lnx}{x}dx = \left.\frac{(Lnx)^2}{x}\right|_1^e = 1/2$, Var(3X) = 9 Var(X), $Var(X) = E(X^2) - [E(X)]^2$

$E(X) = \int_1^e dx = \left. x\right|_1^e = e-1$, $E(X^2) = \int_1^e x\,dx = \left.\frac{x^2}{2}\right|_1^e = \frac{e^2-1}{2}$

$f(x) = cx^2$, $0 < x < 2$, find F(X), P(½ < x < 3/2).

$c\int_0^2 x^2\,dx = 1 \Rightarrow c = 3/8$, $F(X) = \int_0^x \frac{3x^2}{8}dx = \frac{x^3}{8}$, P(½ < x < 3/2) = F(3/2) – F(1/2) = $\frac{27}{64} - \frac{1}{64} = \frac{26}{64}$

$f(x) = c\,x^3$, $-1 < x < 2$, find E(|x|).

$c\int_{-1}^2 x^3\,dx = 1 \Rightarrow c = 4/15$, $E(|x|) = -\int_{-1}^0 \frac{4x^4}{15}dx + \int_0^2 \frac{4x^4}{15}dx = -\left.\frac{4x^5}{75}\right|_{-1}^0 + \left.\frac{4x^5}{75}\right|_0^2 = -\frac{4}{75} + \frac{128}{75} = \frac{124}{75}$

$f(x) = cx^n$, $0 < x < 1$, find P(X > x).

$c\int_0^1 x^n\,dx = 1 \Rightarrow c = n+1$, $P(X > x) = 1 - P(X < x) = 1 - (n+1)\int_0^x x^n\,dx = 1 - x^{n+1}$

$f(x,y) = cxy$, $0 < x < 4$, $1 < y < 5$, find E(X), Var(Y).

$c\int_1^5 (\int_0^4 xy\,dx)\,dy = 1 \Rightarrow c\int_1^5 \left.\frac{x^2y}{2}\right|_0^4 dy = 1 \Rightarrow c\int_1^5 8y\,dy = 1 \Rightarrow c = 1/96$, $f_1(x) = \int_1^5 \frac{xy}{96}dy = \left.\frac{xy^2}{192}\right|_1^5 = x/8$, E(X) =

$\int_0^4 x\,f_1(x)\,dx = \int_0^4 \frac{x^2}{8}dx = \left.\frac{x^3}{24}\right|_0^4 = 8/3$, $f_2(y) = \int_0^4 \frac{xy}{96}dx = \left.\frac{x^2y}{192}\right|_0^4 = y/12$, $E(Y) = \int_1^5 y\,f_2(y)\,dy = \int_1^5 \frac{y^2}{12}dy =$

$\left.\frac{y^3}{36}\right|_1^5 = 31/9$, $E(Y^2) = \int_1^5 y^2\,f_2(y)\,dy = \int_1^5 \frac{y^3}{12}dy = \left.\frac{y^4}{48}\right|_1^5 = \frac{624}{48} = 13$, $Var(Y) = E(Y^2) - [E(Y)]^2 = 92/81$

If a r.v. X takes the values 1,2,3,4 such that 2P(X = 1) = 3P(X=2) = P(X=3) = 5P(X = 4). Find the probability distribution of X.
Let 2P(X = 1) = 3P(X=2) = P(X=3) = 5P(X = 4) = P, therefore P(X = 1) = P/2, P(X =2) = P/3, P(X=3) = P, P(X = 4) = P/5, but P(X = 1) + P(X=2) + P(X=3) + P(X = 4) =1, thus P/2+ P/3+ P+ P/5 =1, P=30/61, then the probability distribution of X is expressed by

X	1	2	3	4
P(X)	15/61	10/61	30/61	6/61

Find the variance and the standard deviation of the sum obtained in tossing a pair of fair dice.
Let the random variable X = sum of the numbers facing up, so the probability distribution can be expressed in the form

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

E(X) = 2(1/36) + 3(2/36) + 4(3/36) + 5(4/36) + 6(5/36)+7(6/36) + 8(5/36) + 9(4/36) + 10(3/36) + 11(2/36) + 12(1/36) = 252/36 = 7 and $E(X^2)=2^2(1/36) + 3^2(2/36) + 4^2(3/36) + 5^2(4/36) + 6^2(5/36) + 7^2(6/36) + 8^2(5/36) + 9^2(4/36) + 10^2(3/36) + 11^2(2/36) + 12^2(1/16) = 1974/36$, therefore: $Var(X) = E(X^2) – (E(X))^2$ = 1974/36 – 49 = 210/36 and standard deviation $\sigma_x = \sqrt{Var(x)}$

Find the variance and the standard deviation of the maximum of the 2 scores obtained in tossing a pair of fair dice.

Let the random variable X = the maximum of the 2 scores, so the probability distribution can be expressed in the form

X	1	2	3	4	5	6
P(X)	1/36	3/36	5/36	7/36	9/36	11/36

E(X) = 1(1/36) + 2(3/36) + 3(5/36) + 4(7/36) + 5(9/36) + 6(11/36) = 161/36 and $E(X^2) = 1^2(1/36) + 2^2(3/36) + 3^2(5/36) + 4^2(7/36) + 5^2(9/36) + 6^2(11/36) = 791/36$, therefore: $Var(X) = E(X^2) – (E(X))^2$ = $791/36 – (161/36)^2 = 2555/1296 \approx 2$ and standard deviation $\sigma_x = \sqrt{Var(x)}$

Find the variance and the standard deviation of the minimum of the 2 scores obtained in tossing a pair of fair dice.

Let the random variable X = the minimum of the 2 scores, so the probability distribution can be expressed in the form:

X	1	2	3	4	5	6
P(X)	11/36	9/36	7/36	5/36	3/36	1/36

$E(X) = 1(11/36)+2(9/36) + 3(7/36) + 4(5/36) + 5(3/36)+6(1/36) = 91/36$ and $E(X^2) = 1^2(11/36) +2^2(9/36) + 3^2(7/36) + 4^2(5/36) + 5^2(3/36) + 6^2(1/36) = 301/36$, therefore: $Var(X) = E(X^2) – (E(X))^2 = 301/36 - (91/36)^2 = = 2555/1296 \approx 2$

Let the r.v. X represents the number of boys in a family of 3 children, find the mean and standard deviation of X.

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

$E(X) = 1(3/8) + 2(3/8) + 3(1/8) = 3/2$, $E(X^2) = 1(3/8) + 4(3/8) + 9(1/8) = 3$, $Var(X) = E(X^2) – (E(X))^2 = 3 – (3/2)^2 = 0.75$ and standard deviation $\sigma_x = \sqrt{Var(x)}$

Four coins are tossed once. Let X be the number of heads obtained, find its mean and standard deviation.

X	0	1	2	3	4
P(X)	1/16	4/16	6/16	4/16	1/16

$E(X) = 1(1/4) + 2(3/8) + 3(1/4) + 4(1/16) = 2$, $E(X^2) = 1(1/4) + 4(3/8) + 9(1/4) +16(1/16) = 5$, $Var(X) = E(X^2) – (E(X))^2 = 5 – (2)^2 = 1$ and standard deviation $\sigma_x = \sqrt{Var(x)} = 1$

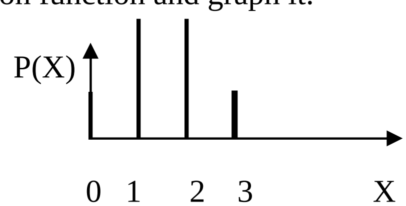
Find the probability distribution of boys and girls in families with 3 children assuming equal probabilities for boys and girls, and then graph the distribution. Also find distribution function and graph it.

Let the r.v. X represents the number of boys in a family of 3 children

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

The distribution function is represented by

X	0	1	2	3
P(X)	1/8	1/2	7/8	1



A random variable X has density function $f(x) = \dfrac{c}{(1+x^2)}$ $\quad -\infty < x < \infty$ .Find the constant c and the probability between 1/3 and 1, then find distribution function.

$c\int_{-\infty}^{\infty} \dfrac{dx}{(1+x^2)} = 1 \Rightarrow c\tan^{-1}x\Big|_{-\infty}^{\infty} = 1 \Rightarrow c = 1/\pi$, $P(1/3 < x < 1) = \int_{1/3}^{1} \dfrac{dx}{\pi(1+x^2)} = \dfrac{\tan^{-1}(1) - \tan^{-1}(1/3)}{\pi}$, $F(X) = \int_{-\infty}^{x} \dfrac{dx}{\pi(1+x^2)} = \dfrac{\tan^{-1}(x) + \pi/2}{\pi}$

Suppose that you are offered the following "deal." You roll a die. If you roll a 6, you win \$10. If you roll a 4 or 5, you win \$5. If you roll a 1, 2, or 3, you pay \$6. Construct a PDF and the expected average winnings per game?

Let the r.v. X represents the number of winning dollars

X	5	6	10
P(X)	1/3	1/2	1/6

$E(X) = 5(1/3) + 6(1/2) + 10(1/6) = 19/3$